

Structuring Mathematical Proofs

A step-by-step framework for setting out logical, coherent algebraic and geometric proofs.

The Foundation of Proof

A formal mathematical proof requires you to demonstrate that a statement is true for *all possible cases*, not just a few examples. Writing "3 is odd, and 3^2 is 9, which is odd" does not prove that the square of all odd numbers is odd. You must use algebra to generalize.

The Essential Vocabulary of Algebraic Proof

To begin any algebraic proof, you must define your variables properly using an integer n :

Concept	Algebraic Translation	Why it works
Any even number	$2n$	Multiplying any integer by 2 guarantees it is a multiple of 2 (even).
Any odd number	$2n + 1$ (or $2n - 1$)	An odd number is always exactly one step away from an even number.
Consecutive integers	n , $n + 1$, $n + 2$	Numbers directly following one another step up by exactly 1.
Consecutive even numbers	$2n$, $2n + 2$, $2n + 4$	Even numbers step up by exactly 2.

The 4-Step Proof Framework

Example Question: Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

1. **State / Define:** Write down your starting variables.

Let the first even number be $2n$. Let the second be $2n + 2$.

2. **Execute the Operations:** Translate the English into algebra.

"Sum of their squares" $\rightarrow (2n)^2 + (2n + 2)^2$

3. **Expand and Simplify:** Do the mechanics.

$$= 4n^2 + (4n^2 + 8n + 4)$$

$$= 8n^2 + 8n + 4$$

4. **Factorise and Conclude:** Factor out the target number (in this case, 4) to prove divisibility, and write a concluding sentence.

$$= 4(2n^2 + 2n + 1)$$

Since 4 is a factor of the expression, the result is a multiple of 4 for all integer values of n .

Structuring Geometric Proofs (Circle Theorems)

When proving angles in a circle, the algebra is usually simpler, but the **C1 (Communication)** marks are strictly enforced. Every single time you calculate a new angle, you must write the exact geometric reason in brackets next to it. For example:

- Angle ABC = 90° (Angles in a semicircle are 90°)
- Angle DEF = Angle DGF (Angles subtended by the same arc at the circumference are equal)

Do not abbreviate the theorems. Writing "Bowtie rule" or "Arrowhead theorem" will result in zero marks. Learn the formal definitions.